The fan is on the same circuit with the electric lights that illuminate the setting circles of the telescope. As a consequence the fan is in operation at frequent intervals throughout the night. As the lights must be turned off whenever an exposure is being made, the fan is out of operation and cannot cause the tube to vibrate at any time that this could be harmful.

These devices have proved successful in removing the aberrations due to temperature. The Hartmann test has since been applied on a number of occasions and has invariably indicated an excellent figure for the objective.

It is not to be supposed that the Thaw objective is peculiar in its behavior with respect to changes in temperature. An examination of the literature on this subject with regard to other objectives reveals the fact that some of them, at least, are similarly affected and exhibit aberrations that change from time to time, precisely as the Thaw objective did before the application of the ventilating devices. It therefore appears to me that the installation of similar arrangements in the case of other telescopes might bring about a considerable improvement in their performance.

The details of this investigation are to appear in Volume 4 of the Publications of this Observatory.

THE RELATIONS BETWEEN THE PROPER MOTIONS AND THE RADIAL VELOCITIES OF THE STARS OF THE SPECTRAL TYPES F, G, K, AND M

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Among the stars observed for radial velocity with the 60-inch reflector of the Mount Wilson Observatory, there are many of the spectral types *F*, *G*, *K*, and *M* which have either very large or very small astronomical proper motions. The two classes of stars show some very marked differences which indicate a close relationship between proper motion and radial velocity. It is the aim of the present note to bring out this relationship as clearly as possible.

For this purpose it is necessary to supplement the Mount Wilson data for the stars of extreme proper motion with similar data for the rest of the stars. There is, of course, no better source for this purpose than the catalogue of radial velocities published recently by the Lick Observa-

tory.[1] The Bonn observations by Küstner came to hand too late for any extended use. The classification used is that of the Harvard College Observatory, and the spectra given in Campbell's catalogue are, with a very few exceptions, those of the Harvard Revised Photometry.[2] The Mount Wilson stars have been classified by the method described in the paper by Adams and Kohlschütter.[3] Occasionally large differences have been found from the estimates of the Harvard observers for the same stars.

The observed radial motions were first reduced to velocities (ρ) relative to the center of gravity of the stellar system by applying the component of the sun's motion through space with reversed sign; that is, the correction $V \cos \theta$, where V is the sun's velocity, and θ the angular distance between star and apex. For the apex we adopted the position α (1900) = 18h, 0m; $\delta = +31°$, and for V the value 20.0 km. per second. The values of ρ thus obtained were grouped: (1) according to amount of proper motion; (2) according to distance λ from the nearest true vertex of the two star-streams. The positions of these vertices, according to the determinations of Kapteyn, Eddington, and others are:

Stream I, 18h 12m $-12°$; Stream II, 6h 12m $+12°$

The results are summarized in Table I. The letter μ denotes the total proper motion, and W and C refer to the radial velocity determinations by the Mount Wilson observers and by Campbell, respectively. Average values are indicated by dashes over the letters, and figures in parentheses denote the number of stars on which each average is based. The two columns under λ are, as a rule, limited by values of λ 60° to 90° ($\bar{\rho}_1$), and 0° to 49° ($\bar{\rho}_2$). A third group between the narrow limits 50° to 59° was formed, but is omitted from the table. In a few cases where the number of stars was small this middle group was used to increase the weight of the extreme groups. Six stars of abnormal velocity have been excluded out of a total of 1106 stars. These are:

Lal. 15290 -249.4	A.Oe 20452 -163.0	Boss 130 -80.7
Boss 315 $+62.8$	Boss 1511 $+163.0$	A.Oe 14320 $+299.4$

An inspection of Table I shows clearly: first, that the average velocity of the stars near the vertices is considerably greater than that of the stars far away from these points; second, that the average value of ρ increases rapidly with the proper motion.

The first of these conclusions is indicated most concisely by the values of the ratio $\bar{\rho}_2/\bar{\rho}_1$ in the last column. It is a necessary consequence of the existence of two star streams with opposite motions. Thus, it is

evident that if all of the stars moved accurately in the direction of these streams the radial velocity would be zero at those points where the line

TABLE I

AVERAGE VALUES OF THE RADIAL VELOCITY ρ IN KM.

TYPE	PROPER MOTION	AUTHORITY	λ 60° to 90°			λ 0° to 49°			TOTAL			$\bar{\rho}_2/\bar{\rho}_1$	
	μ		$\bar{\lambda}$	$\bar{\rho}_1$		$\bar{\lambda}$	$\bar{\rho}_2$		$\bar{\lambda}$	$\bar{\rho}$		Obs.	Theory
F	0.000″ to 0.029″	W+C	79°	8.9	(25)	32°	13.6	(22)	56°	10.6	(54)	1.53	1.63
	0.030 0.069	C	71	8.5	(9)	29	12.5	(11)	48	10.15	(22)	1.47	1.56
	0.070 0.149	C	78	12.3	(17)	34	20.5	(17)	53	14.65	(38)	1.67	1.58
	0.150 0.249	C	72	11.5	(22)	32	18.9	(12)	58	14.3	(40)	1 64	1.55
	0.250 0.499	C	74	16.7	(24)	33	21.1	(8)	62	16.2	(38)	1.26	1.54
	≧0.500	W+C	73	18.05	(30)	43	34.9	(18)	62	24.4	(48)	1.93	1.39
G	0.000 to 0.026	W+C	73	6.9	(37)	32	12.8	(32)	54	9.6	(80)	1.86	1.55
	0.027 0.049	C	76	9.6	(10)	37	12.8	(13)	54	11.8	(28)	1.33	1.53
	0.050 0.099	C	74	9.5	(18)	27	12.3	(7)	61	10.3	(25)	1.30	1.63
	0.100 0.499	C	76	14.95	(13)	44	23.6	(11)	60	19.0	(28)	1.58	1.41
	≧0.500	W+C	76	22.9	(51)	37	45.6	(31)	60	32.1	(91)	1.99	1.51
K	0.000 to 0.025	W+C	76	10.9	(47)	36	12.3	(39)	57	11.7	(91)	1.13	1.52
	0.026 0.039	C	73	12.15	(37)	37	11.9	(24)	58	11.7	(68)	0.98	1.47
	0.040 0.059	C	75	14.9	(21)	38	21.3	(21)	56	17.9	(50)	1.43	1.49
	0.060 0.079	C	75	10.7	(24)	36	15.1	(16)	58	12.3	(41)	1.41	1.50
	0.080 0.099	C	74	15.7	(26)	34	14.0	(10)	62	15.2	(36)	0.89	1.52
	0.100 0.119	C	76	20.7	(12)	32	22.6	(15)	52	23.4	(31)	1.09	1.58
	0.120 0.149	C	73	16.5	(24)	39	23.8	(10)	62	19.0	(36)	1.44	1.43
	0.150 0.199	C	74	15.6	(24)	39	24.5	(10)	62	18.2	(34)	1.57	1.45
	0.200 0.299	C	75	16.5	(18)	32	24.3	(13)	56	20.4	(37)	1.47	1.56
	0.300 0.599	C	73	33.3	(12)	33	47.05	(7)	58	38.35	(19)	1.41	1.52
	≧0.600	W+C	74	23.5	(25)	34	23.0	(28)	53	22.5	(58)	0.98	1.52
M	0.000 to 0.029	W+C	76	12.9	(18)	36	18.1	(15)	58	14.4	(40)	1.40	1.52
	0.030 0.089	C	79	15.0	(19)	36	21.0	(11)	62	17.0	(34)	1.40	1.55
	0.090 0.499	C	77	17.9	(14)	35	28.9	(7)	62	20.7	(24)	1.61	1.56
	≧0.500	W	71	53.2	(4)				71	53.2	(4)		

of sight is at right angles to the stream motion; that is, where $\lambda = 90°$. On the other hand the motion would be altogether radial at the vertices themselves; that is, ρ should be large for $\lambda = 0°$. Since the motions of the stars do not in general coincide with the stream motion, but only show a preference for directions differing but little from it, the contrast in the amount of the radial motion at $\lambda = 90°$ and $\lambda = 0°$ will be less marked. This amount can, of course, be calculated as soon as we have derived complete elements for the two streams from a consideration of the proper motions. As it is, different theories give slightly different values,[4] and eventually these differences may serve to determine the theory to be

preferred. If we use that developed by Kapteyn[5] we obtain the comparison shown in Table I. Except for some of the K-type stars the agreement of the values is very satisfactory. The theory, of course, requires that ρ be proportional to $1 + \cos^2 \lambda$.

The values of $\bar{\rho}_2/\bar{\rho}_1$ are no smaller for the stars having the very smallest proper motions than for the other stars in the list. This proves that the two star streams extend to the greatest distances for which we have means of judging. This fact has been doubted by Eddington[6] who proposed an attractive explanation of the behavior of the helium stars, which show hardly a trace of the second stream, on the supposition that the greater part of them are beyond the region where we must admit the existence of two streams. Several objections have already been raised against this hypothesis,[7] and we believe that Eddington has abandoned it. It is interesting, however, to find in the radial motions of these stars so strong a proof of the great extent of the star streams.

The low value of $\bar{\rho}_2/\bar{\rho}_1$ for some of the K stars, particularly for some of those of very small proper motion, seems difficult of explanation. A preliminary investigation of the proper motions for the stars having values between 0.026″ and 0.039″ indicates that members of the second stream are not wanting. Evidently the anomally will require much further study.

The second conclusion indicated by Table I, that ρ increases with increase in proper motion, may be explained in any one of at least three ways. We may assume that:

a. The real velocity of the stars decreases with the distance.

b. The more luminous stars move more slowly than the fainter ones.

c. The distribution of the velocities of the stars is not in accordance with Maxwell's law, the large velocities being in excess.

The application of the first two of these explanations is evident. In the consecutive groups for each spectral type the average magnitude is roughly the same. Hence, with the average proper motions increasing regularly from one group to another, the average distances must decrease, and the luminosities become less.

The possibility of the third explanation, *c*, may seem somewhat less evident. Its consideration, however, is essential since it has already been shown by Schwarzschild[8] that the distribution of velocities, using values relative to the sun, cannot agree with Maxwell's law, and that the larger values must be in excess. The radial velocities used here afford the data for the derivation of the velocity-law relative to the center of the stellar system. Such a derivation we have actually carried out in the

case of the K stars, and we hope in the course of a more extended discussion to describe it in detail, as well as to obtain similar expressions for the stars of other types. A very brief summary of the method used is as follows:

1. All of the stars between the limits $\lambda = 60°$ and $\lambda = 90°$ were selected from Campbell's catalogue and arranged according to amount of radial velocity. This selection was made in order to eliminate the effect of stream motion so far as possible. The application of Maxwell's law to this material at once showed a large preponderance of high velocities. Thus there are 17 stars with velocities above 40 km. per second where Maxwell's law requires less than one-third of this number.

2. A satisfactory expression for representing the observed distribution of velocities was found in the sum of two Maxwell equations with different moduli. A peculiar feature of this result is that if all of the stars are used, and not alone those on which the stream motion has little influence, the exponential constants remain essentially unchanged.

3. With the aid of this expression the relationship was determined between the average radial velocity and the component of the linear velocity at right angles to the line of sight.

4. The stars were then divided into groups, each within a narrow range of proper motion, so that they may all be regarded as at the same distance. The components of the linear velocities at right angles to the line of sight were then computed by the aid of the parallaxes given in *Groningen Publications* No. 8. A factor was applied to the parallaxes such as to make the total average linear velocity for all of the stars equal to the total average radial velocity ρ. Table II shows the final results for ρ.

It appears from these results, therefore, that our assumption c is quite adequate to explain the variation of radial velocity with proper motion. There remains, however, the possible effect of a or b, which we may designate briefly as the effect of distance or of absolute magnitude. With more extensive data it might perhaps be possible to separate these two effects. As it is we shall suppose the distance effect to be negligible and try to determine the absolute magnitude effect.

For stars of approximately the same distance we shall assume that $\rho = a + b\,M$, where M is the absolute magnitude derived from the formula,[9] $M = m + 5 + 5 \log \pi$. In general we have kept the same groups used in Table I, the variation in distance not being excessive, but in a few cases consecutive groups have been combined. These groups were then further divided according to apparent magnitudes, an attempt being made to keep the numbers of the stars in the extreme subdivisions about equal. The values of the parallax π were then taken from *Gronin-*

gen Publications No. 8, and the absolute magnitudes were computed. From the resulting equations the values of b were determined by least squares.

TABLE II

COMPUTATION OF THE RADIAL VELOCITY AS A FUNCTION OF THE PROPER MOTION

PROPER MOTION μ	NO. STARS CAMPBELL	PARALLAX π GRON. NO. 8	$\bar{\rho}$ OBS.	$\bar{\rho}$ COMP.	DIFFERENCE
seconds (″)		*seconds* (″)	*km.*	*km.*	*km.*
0.000 to 0.025	27	0.0064	10.9[1]	12.1	−1.2
0.026 0.039	37	0.0112	12.15	12.5	−0.35
0.040 0.059	21	0.0148	14.9	12.9	+2.0
0.060 0.079	24	0.0185	10.7	13.3	−2.6
0.080 0.099	26	0.0218	15.7	13.7	+2.0
0.100 0.119	12	0.0248	20.7	14.3	+6.4
0.120 0.149	24	0.0283	16.5	14.8	+1.7
0.150 0.199	24	0.0332	15.6	15.9	−0.3
0.200 0.299	18	0.0412	16.5	17.7	−1.2
$\leqq$0.300	19	0.108	26.7[1]	24.5	+2.2
	232				

[1] These numbers have the Mount Wilson results for the corresponding proper motions included in them.

This solution requires two corrections. The first is due to the difference in the average value of λ for the several groups. This correction has been applied on the very probable assumption that the change of ρ with λ is proportional to $1 + \cos^2 \lambda$. The second correction is due to the fact that while the values of μ are equal, or practically so, the values of π are somewhat unequal owing to the differences in magnitude. As a result the average linear motions and hence the radial velocities will be affected. This effect is systematic in character, though small, and can be applied only in the case of the K stars.

If we combine the results for the separate groups into means for each spectral type we obtain Table III. The corrected value for the K stars is denoted by $b2$. On the assumption, which seems probable, that the correction in the case of the other spectra is the same as for the K stars we find, therefore, *a change of radial velocity per unit of absolute magnitude of* 1.1 km. $\pm$ 0.4.

TABLE III. VALUES OF THE CONSTANT b IN KM.

TYPE	$b1$	$b2$	PROB. ERROR
F	+2.1	.....	$\pm$0.73
G	+1.0	.....	0.82
K	+1.6	+1.1	0.62
M	+2.6	.....	2.5
Mean	+1.6	.....	$\pm$0.40

The reality of this conclusion that the more luminous stars move more

slowly than the fainter stars must be accepted with considerable reserve. Certain features of this investigation are necessarily somewhat crude, and the data upon which it rests were not collected with a view to such a question. Moreover, of two direct observational points of evidence one appears to be opposed to this conclusion.

The stars denoted as *c* stars by Miss Maury were shown by Hertzsprung to be exceedingly luminous and very distant. According to an unpublished investigation by Kapteyn and Hertzsprung these stars, of average magnitude 4.5, must be 300 times more luminous and 4.5 times as far away as the average *A* stars of the fifth magnitude. We should, therefore, expect a low velocity in their case. In reality the average radial velocity of 28 of these stars is 12.8 km. For the average of all the *A* stars Campbell finds 11.1 km. In considering this result it should be borne in mind that the effect found here is for the second type stars and may not apply to the *A* stars; also that the *c* stars may actually constitute a separate spectral class with which the *A* stars are not directly comparable.

It is known from the researches of Hertzsprung that the variable stars of the δ *Cephei* type are stars of very high luminosity. He finds for their average absolute magnitude the value -2.2, using for this purpose their parallactic motions and apparent magnitudes. The average absolute magnitude of the 198 *F* stars given by Campbell is $+1.8$. The average radial velocity, freed from the sun's motion, of 11 of the δ *Cephei* variables, of average type *F*8, is 9.0 ± 1.2 km. The average radial velocity of Campbell's *F* stars is 14.75 km. This would give 1.5 km. as the value for our coefficient *b*.

SUMMARY OF RESULTS

1. The radial velocities furnish a very thorough test of the theory of the star streams. The results found for the *F*, *G*, *K*, and *M* stars are in close agreement with those we should expect from the theory as derived from proper motions.

2. The radial velocities of the stars of the smallest proper motions show the effects of the two star-streams with the same certainty as those of the other stars. The existence of the two star-streams is, therefore, proved at the greatest distances for which we have adequate data.

3. The *K* stars behave in general like the other stars, but there are a few exceptional cases. These do not appear to be due to the absence of the second stream.

4. For all of the spectral classes the average radial velocities show a regular increase with the proper motion.

5. Such a change of radial velocity is a necessary consequence of a velocity distribution (for the peculiar motions) different from that given by Maxwell's law.

6. A first approximation to the velocity distribution has been derived for the *K* stars. It explains the change of velocity with proper motion in a satisfactory manner.

7. Some positive indications have been found of a change of radial velocity with absolute magnitude, the brighter stars moving more slowly than the fainter stars.

[1] *Lick Obs. Bull.*, 7, 113 (1913).

[2] See *Annals Harvard Coll. Obs.*, vol. **50**.

[3] Adams and Kohlschütter, "Some special criteria for the determination of absolute stellar magnitudes," *Contrib. Mt. Wilson Solar Obs.*, No. 89; *Astrophys. J.* **40** (1914).

[4] For example, *Astrophysical Journal*, **31**, 266 (1910).

[5] Kapteyn, *Monthly Notices*, 72, 743 (1912).

[6] Eddington, *Observatory*, 34, 355 (1911).

[7] *Proc. Amsterdam Acad. Sciences*, 14, 524 (1911).

[8] Schwarzschild, *Astron. Nachr.*, No. 4557 (1912).

[9] See *Publ. Astron. Lab. Groningen*, No. 11, p. 12.

CRITIQUE OF THE HYPOTHESIS OF ANOMALOUS DISPERSION IN CERTAIN SOLAR PHENOMENA

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According to the theory of anomalous dispersion, the interpretations of solar observations in terms of absorption, motion, pressure, and level are generally misleading since the observed phenomena, such as prominences, flash spectra, flocculi, and displacements of the Fraunhofer lines are from this point of view mainly the effects of anomalous refraction in the solar atmosphere, so that in their study we are facing optical illusions. The establishment of such a point of view would revolutionize or render futile many of the present lines of solar and stellar observation, and would make practically impossible the solution of many problems which confront the investigator. It is therefore of the highest importance to determine the form and extent of the influence of anomalous refraction in the solar atmosphere, if it obtains to a detectible amount.

There is a degree of vagueness in the deductions from the theory, due to its extreme flexibility, that makes a quantitative examination of its claims difficult. During the present year, however, W. H. Julius, its